CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Expand $f(x) = x - x^2$ as a Fourier series in the interval $(-\pi, \pi)$. (08 Marks)

b. Obtain the half range Fourier cosine series for the function $f(x) = \sin x$, $0 \le x \le \pi$. (06 Marks)

c. Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier series of f(x) as given in the following table:

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

(06 Marks)

OR

2 a. Obtain the Fourier series for $f(x) = \sin mx$ in the range $(-\pi, \pi)$ where m is neither zero nor an integer. (08 Marks)

b. Obtain half range cosine series for

$$f(x) = \begin{cases} Kx, & 0 \le x \le \ell/2 \\ K(\ell - x), & \ell/2 \le x \le \ell \end{cases}$$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$ (06 Marks)

c. Express Y as a Fourier series upto first harmonic, given that

X	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
Y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

(06 Marks)

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx.$$ (08 Marks)

b. Obtain the Fourier cosine transform of e^{-ax}. (06 Marks)

c. Obtain the inverse Z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (06 Marks)

OR

4 a. Given $Z(u_n) = \frac{2z^2 + 3z + 4}{(z-3)^3}$; $|z| > 3$, show that $u_0 = 0$, $u_1 = 2$, $u_2 = 21$. (08 Marks)

b. Solve by using Z-transforms, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$. (06 Marks)

c. Find the Fourier transform of $f(x) = e^{-|x|}$ (06 Marks)

Module-3

5 a. Find the coefficient of correlation for the following data:

x	50	50	55	60	65	65	65	60	60	50
y	11	13	14	16	16	15	15	14	13	13

(08 Marks)

b. By the method of least square, find the straight line that best fits the following data:

x	1	2	3	4	5
y	14	27	40	55	68

(06 Marks)

c. Use Newton-Raphson method to find a root of the equation $\tan x - x = 0$ near $x = 4.5$ (x is in radians) carry out three iterations. (06 Marks)

OR

6 a. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(08 Marks)

b. Fit a second degree parabola to the following data:

x	1	2	3	4	5
y	10	12	13	16	19

(06 Marks)

c. Find the real root of the equation $xe^x - 3 = 0$ by Regula-Falsi method, correct to three decimal places in (1, 2). (06 Marks)

Module-4

7 a. From the following table find f(86) using Newton's backward interpolation formula:

x	40	50	60	70	80	90
f(x)	180	204	226	250	276	304

(08 Marks)

b. Given the values:

x	5	7	11	13	17
f(x)	150	392	1452	2360	5202

Evaluate f(9), using Newton's divided difference formula. (06 Marks)

c. Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x)dx$ using Simpson's $\frac{3}{8}^{th}$ rule taking six equal parts. (06 Marks)

OR

8 a. Find an approximate value of f(x) at x = 1.1 from the following data:

x	1	1.2	1.4	1.6	1.8	2
f(x)	0	0.128	0.544	1.296	2.432	4

(08 Marks)

b. Find the polynomial f(x) by using Langrage's formula from the following data:

x	0	1	2	5
f(x)	2	3	12	147

(06 Marks)

c. Evaluate $\int_{4}^{5.2} \log_e x dx$ by Weddle's rule taking six equal strips. (06 Marks)

Module-5

9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(08 Marks)**

b. Evaluate $\int_C xydx + xy^2dy$ by Stoke's theorem where c is the square in the x-y plane with vertices $(1, 0)$ $(-1, 0)$ $(0, 1)$ $(0, -1)$. **(06 Marks)**

c. Derive Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$ **(06 Marks)**

OR

10 a. If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, evaluate $\iiint_V \nabla \cdot \vec{F}\,dv$ where v is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$. **(08 Marks)**

b. Find the extremal of the functional $\int_{x_1}^{x_2} \left[(y')^2 - y^2 + 2y\sec x\right]dx$. **(06 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them. **(06 Marks)**

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